Branching random walks and Gaussian fields

Ofer Zeitouni

Weizmann Institute
and
University of Minnesota

June 2012

Plan of Lectures

Lectures I,II - Branching random walks in Z - maximal displacement

- Model
- Law of large numbers: comparison with independent walks, first and second moments.
- The Dekking-Host argument and tightness.
- Logarithmic corrections for the mean - the lower bound.

Lecture III - Maximal displacement - ct'd

- Logarithmic corrections - upper bound.
- Time varying profiles. Phase transitions and non-logarithmic corections.

Lectures IV,V - the 2D Gaussian free field

- Model and Markov property; maxima.
- Basic inequalities: Slepian, Borell-Tsirelson, Sudokov-Fernique.
- LLN for maxima - upper bound and approximate tree structure.
- The Dekking-Host argument adapted.
- Expectation of maximum: comparison and modified BRW.
- Analysis of MBRW

Plan of Lectures

Lectures I,II - Branching random walks in Z - maximal displacement

- Model
- Law of large numbers: comparison with independent walks, first and second moments.
- The Dekking-Host argument and tightness.
- Logarithmic corrections for the mean - the lower bound.

Lecture III - Maximal displacement - ct'd

- Logarithmic corrections - upper bound.
- Time varying profiles. Phase transitions and non-logarithmic corections.

Lectures IV,V - the 2D Gaussian free field

- Model and Markov property; maxima.
- Basic inequalities: Slepian, Borell-Tsirelson, Sudokov-Fernique.
- LLN for maxima - upper bound and approximate tree structure.
- The Dekking-Host argument adapted.
- Expectation of maximum: comparison and modified BRW.
- Analysis of MBRW

Plan of Lectures

Lectures I,II - Branching random walks in Z - maximal displacement

- Model
- Law of large numbers: comparison with independent walks, first and second moments.
- The Dekking-Host argument and tightness.
- Logarithmic corrections for the mean - the lower bound.

Lecture III - Maximal displacement - ct'd

- Logarithmic corrections - upper bound.
- Time varying profiles. Phase transitions and non-logarithmic corections.

Lectures IV,V - the 2D Gaussian free field

- Model and Markov property; maxima.
- Basic inequalities: Slepian, Borell-Tsirelson, Sudokov-Fernique.
- LLN for maxima - upper bound and approximate tree structure.
- The Dekking-Host argument adapted.
- Expectation of maximum: comparison and modified BRW.
- Analysis of MBRW

Plan of Lectures

Lectures I,II - Branching random walks in Z - maximal displacement

- Model
- Law of large numbers: comparison with independent walks, first and second moments.
- The Dekking-Host argument and tightness.
- Logarithmic corrections for the mean - the lower bound.

Lecture III - Maximal displacement - ct'd

- Logarithmic corrections - upper bound.
- Time varying profiles. Phase transitions and non-logarithmic corections.

Lectures IV,V - the 2D Gaussian free field

- Model and Markov property; maxima.
- Basic inequalities: Slepian, Borell-Tsirelson, Sudokov-Fernique.
- LLN for maxima - upper bound and approximate tree structure.
- The Dekking-Host argument adapted.
- Expectation of maximum: comparison and modified BRW.
- Analysis of MBRW

Plan of Lectures

Lectures I,II - Branching random walks in Z - maximal displacement

- Model
- Law of large numbers: comparison with independent walks, first and second moments.
- The Dekking-Host argument and tightness.
- Logarithmic corrections for the mean - the lower bound.

Lecture III - Maximal displacement - ct'd

- Logarithmic corrections - upper bound.
- Time varying profiles. Phase transitions and non-logarithmic corections.

Lectures IV,V - the 2D Gaussian free field

- Model and Markov property; maxima.
- Basic inequalities: Slepian, Borell-Tsirelson, Sudokov-Fernique.
- LLN for maxima - upper bound and approximate tree structure.
- The Dekking-Host argument adapted.
- Expectation of maximum: comparison and modified BRW.
- Analysis of MBRW

Plan of Lectures

Lectures I,II - Branching random walks in Z - maximal displacement

- Model
- Law of large numbers: comparison with independent walks, first and second moments.
- The Dekking-Host argument and tightness.
- Logarithmic corrections for the mean - the lower bound.

Lecture III - Maximal displacement - ct'd

- Logarithmic corrections - upper bound.
- Time varying profiles. Phase transitions and non-logarithmic corections.

Lectures IV,V - the 2D Gaussian free field

- Model and Markov property; maxima.
- Basic inequalities: Slepian, Borell-Tsirelson, Sudokov-Fernique.
- LLN for maxima - upper bound and approximate tree structure.
- The Dekking-Host argument adapted.
- Expectation of maximum: comparison and modified BRW.
- Analysis of MBRW

Plan of Lectures

Lectures I,II - Branching random walks in Z - maximal displacement

- Model
- Law of large numbers: comparison with independent walks, first and second moments.
- The Dekking-Host argument and tightness.
- Logarithmic corrections for the mean - the lower bound.

Lecture III - Maximal displacement - ct'd

- Logarithmic corrections - upper bound.
- Time varying profiles. Phase transitions and non-logarithmic corections.

Lectures IV,V - the 2D Gaussian free field

- Model and Markov property; maxima.
- Basic inequalities: Slepian, Borell-Tsirelson, Sudokov-Fernique.
- LLN for maxima - upper bound and approximate tree structure.
- The Dekking-Host argument adapted.
- Expectation of maximum: comparison and modified BRW.
- Analysis of MBRW

Plan of Lectures

Lectures I,II - Branching random walks in Z - maximal displacement

- Model
- Law of large numbers: comparison with independent walks, first and second moments.
- The Dekking-Host argument and tightness.
- Logarithmic corrections for the mean - the lower bound.

Lecture III - Maximal displacement - ct'd

- Logarithmic corrections - upper bound.
- Time varying profiles. Phase transitions and non-logarithmic corections.

Lectures IV,V - the 2D Gaussian free field

- Model and Markov property; maxima.
- Basic inequalities: Slepian, Borell-Tsirelson, Sudokov-Fernique.
- LLN for maxima - upper bound and approximate tree structure.
- The Dekking-Host argument adapted.
- Expectation of maximum: comparison and modified BRW.
- Analysis of MBRW

Plan of Lectures

Lectures I,II - Branching random walks in Z - maximal displacement

- Model
- Law of large numbers: comparison with independent walks, first and second moments.
- The Dekking-Host argument and tightness.
- Logarithmic corrections for the mean - the lower bound.

Lecture III - Maximal displacement - ct'd

- Logarithmic corrections - upper bound.
- Time varying profiles. Phase transitions and non-logarithmic corections.

Lectures IV,V - the 2D Gaussian free field

- Model and Markov property; maxima.
- Basic inequalities: Slepian, Borell-Tsirelson, Sudokov-Fernique.
- LLN for maxima - upper bound and approximate tree structure.
- The Dekking-Host argument adapted.
- Expectation of maximum: comparison and modified BRW.
- Analysis of MBRW

Plan of Lectures

Lectures I,II - Branching random walks in Z - maximal displacement

- Model
- Law of large numbers: comparison with independent walks, first and second moments.
- The Dekking-Host argument and tightness.
- Logarithmic corrections for the mean - the lower bound.

Lecture III - Maximal displacement - ct'd

- Logarithmic corrections - upper bound.
- Time varying profiles. Phase transitions and non-logarithmic corections.

Lectures IV,V - the 2D Gaussian free field

- Model and Markov property; maxima.
- Basic inequalities: Slepian, Borell-Tsirelson, Sudokov-Fernique.
- LLN for maxima - upper bound and approximate tree structure.
- The Dekking-Host argument adapted.
- Expectation of maximum: comparison and modified BRW.
- Analysis of MBRW

Plan of Lectures

Lectures I,II - Branching random walks in Z - maximal displacement

- Model
- Law of large numbers: comparison with independent walks, first and second moments.
- The Dekking-Host argument and tightness.
- Logarithmic corrections for the mean - the lower bound.

Lecture III - Maximal displacement - ct'd

- Logarithmic corrections - upper bound.
- Time varying profiles. Phase transitions and non-logarithmic corections.

Lectures IV,V - the 2D Gaussian free field

- Model and Markov property; maxima.
- Basic inequalities: Slepian, Borell-Tsirelson, Sudokov-Fernique.
- LLN for maxima - upper bound and approximate tree structure.
- The Dekking-Host argument adapted.
- Expectation of maximum: comparison and modified BRW.
- Analysis of MBRW

Plan of Lectures

Lectures I,II - Branching random walks in Z - maximal displacement

- Model
- Law of large numbers: comparison with independent walks, first and second moments.
- The Dekking-Host argument and tightness.
- Logarithmic corrections for the mean - the lower bound.

Lecture III - Maximal displacement - ct'd

- Logarithmic corrections - upper bound.
- Time varying profiles. Phase transitions and non-logarithmic corections.

Lectures IV,V - the 2D Gaussian free field

- Model and Markov property; maxima.
- Basic inequalities: Slepian, Borell-Tsirelson, Sudokov-Fernique.
- LLN for maxima - upper bound and approximate tree structure.
- The Dekking-Host argument adapted.
- Expectation of maximum: comparison and modified BRW.
- Analysis of MBRW

$\mathcal{T}$ - tree rooted at o.
$|v|$ - distance of v from root. d_v - degree of v.
$D_n := \{v \in V : |v| = n\}$ (nth generation).
$o \leftrightarrow v$ - vertices/edges on geodesic connecting o and v.

BRW model

$\{X_e\}_{e \in E}$ i.i.d., law μ.
$S_v = \sum_{e \in o \leftrightarrow v} X_e$ sum along geodesic (BRW).

Maximal displacement:

$$M_n = \max_{v \in D_n} S_v .$$

Assumptions (for these lectures):

μ possesses super-exponential tails:

$$E_\mu(e^{\lambda X_e}) =: e^{\Lambda(\lambda)} < \infty, \quad \lambda \in \mathbb{R}.$$

The tree $\mathcal{T}$ is a k-ary tree, with $k \geq 2$: $d_v = k + 1_{v \neq o}$.

Branching Random Walks

$\mathcal{T}$ - tree rooted at o.
$|v|$ - distance of v from root. d_v - degree of v.
$D_n := \{v \in V : |v| = n\}$ (nth generation).
$o \leftrightarrow v$ - vertices/edges on geodesic connecting o and v.

BRW model

$\{X_e\}_{e \in E}$ i.i.d., law μ.
$S_v = \sum_{e \in o \leftrightarrow v} X_e$ sum along geodesic (BRW).

Maximal displacement:

$$M_n = \max_{v \in D_n} S_v .$$

Assumptions (for these lectures):

μ possesses super-exponential tails:

$$E_\mu(e^{\lambda X_e}) =: e^{\Lambda(\lambda)} < \infty, \quad \lambda \in \mathbb{R}.$$

The tree $\mathcal{T}$ is a k-ary tree, with $k \geq 2$: $d_v = k + 1_{v \neq o}$.

$\mathcal{T}$ - tree rooted at o.
$|v|$ - distance of v from root. d_v - degree of v.
$D_n := \{v \in V : |v| = n\}$ (nth generation).
$o \leftrightarrow v$ - vertices/edges on geodesic connecting o and v.
BRW model
$\{X_e\}_{e \in E}$ i.i.d., law μ.
$S_v = \sum_{e \in o \leftrightarrow v} X_e$ sum along geodesic (BRW).
Maximal displacement:

$$M_n = \max_{v \in D_n} S_v .$$

Assumptions (for these lectures):
μ possesses super-exponential tails:

$$E_\mu(e^{\lambda X_e}) =: e^{\Lambda(\lambda)} < \infty, \quad \lambda \in \mathbb{R}.$$

The tree $\mathcal{T}$ is a k-ary tree, with $k \geq 2$: $d_v = k + 1_{v \neq o}$.

Branching Random Walks

$\mathcal{T}$ - tree rooted at o.
$|v|$ - distance of v from root. d_v - degree of v.
$D_n := \{v \in V : |v| = n\}$ (nth generation).
$o \leftrightarrow v$ - vertices/edges on geodesic connecting o and v.

BRW model

$\{X_e\}_{e \in E}$ i.i.d., law μ.
$S_v = \sum_{e \in o \leftrightarrow v} X_e$ sum along geodesic (BRW).

Maximal displacement:

$$M_n = \max_{v \in D_n} S_v .$$

Assumptions (for these lectures):

μ possesses super-exponential tails:

$$E_\mu(e^{\lambda X_e}) =: e^{\Lambda(\lambda)} < \infty, \quad \lambda \in \mathbb{R}.$$

The tree $\mathcal{T}$ is a k-ary tree, with $k \geq 2$: $d_v = k + 1_{v \neq o}$.

Branching Random Walks

$\mathcal{T}$ - tree rooted at o.
$|v|$ - distance of v from root. d_v - degree of v.
$D_n := \{v \in V : |v| = n\}$ (nth generation).
$o \leftrightarrow v$ - vertices/edges on geodesic connecting o and v.

BRW model

$\{X_e\}_{e \in E}$ i.i.d., law μ.
$S_v = \sum_{e \in o \leftrightarrow v} X_e$ sum along geodesic (BRW).

Maximal displacement:

$$M_n = \max_{v \in D_n} S_v \, .$$

Assumptions (for these lectures):

μ possesses super-exponential tails:

$$E_\mu(e^{\lambda X_e}) =: e^{\Lambda(\lambda)} < \infty, \quad \lambda \in \mathbb{R}.$$

The tree $\mathcal{T}$ is a k-ary tree, with $k \geq 2$: $d_v = k + 1_{v \neq o}$.

Branching Random Walks

$\mathcal{T}$ - tree rooted at o.
$|v|$ - distance of v from root. d_v - degree of v.
$D_n := \{v \in V : |v| = n\}$ (nth generation).
$o \leftrightarrow v$ - vertices/edges on geodesic connecting o and v.

BRW model

$\{X_e\}_{e \in E}$ i.i.d., law μ.
$S_v = \sum_{e \in o \leftrightarrow v} X_e$ sum along geodesic (BRW).

Maximal displacement:

$$M_n = \max_{v \in D_n} S_v .$$

Assumptions (for these lectures):

μ possesses super-exponential tails:

$$E_\mu(e^{\lambda X_e}) =: e^{\Lambda(\lambda)} < \infty, \quad \lambda \in \mathbb{R}.$$

The tree $\mathcal{T}$ is a k-ary tree, with $k \geq 2$: $d_v = k + 1_{v \neq o}$.

$\mathcal{T}$ - tree rooted at o.
$|v|$ - distance of v from root. d_v - degree of v.
$D_n := \{v \in V : |v| = n\}$ (nth generation).
$o \leftrightarrow v$ - vertices/edges on geodesic connecting o and v.

BRW model

$\{X_e\}_{e \in E}$ i.i.d., law μ.
$S_v = \sum_{e \in o \leftrightarrow v} X_e$ sum along geodesic (BRW).

Maximal displacement:

$$M_n = \max_{v \in D_n} S_v \,.$$

Assumptions (for these lectures):

μ possesses super-exponential tails:

$$E_\mu(e^{\lambda X_e}) =: e^{\Lambda(\lambda)} < \infty, \quad \lambda \in \mathbb{R}.$$

The tree $\mathcal{T}$ is a k-ary tree, with $k \geq 2$: $d_v = k + 1_{v \neq o}$.

$\mathcal{T}$ - tree rooted at o.
$|v|$ - distance of v from root. d_v - degree of v.
$D_n := \{v \in V : |v| = n\}$ (nth generation).
$o \leftrightarrow v$ - vertices/edges on geodesic connecting o and v.

BRW model

$\{X_e\}_{e \in E}$ i.i.d., law μ.
$S_v = \sum_{e \in o \leftrightarrow v} X_e$ sum along geodesic (BRW).

Maximal displacement:

$$M_n = \max_{v \in D_n} S_v \,.$$

Assumptions (for these lectures):

μ possesses super-exponential tails:

$$E_\mu(e^{\lambda X_e}) =: e^{\Lambda(\lambda)} < \infty, \quad \lambda \in \mathbb{R}.$$

The tree $\mathcal{T}$ is a k-ary tree, with $k \geq 2$: $d_v = k + 1_{v \neq o}$.

Branching Random Walks

$\mathcal{T}$ - tree rooted at o.
$|v|$ - distance of v from root. d_v - degree of v.
$D_n := \{v \in V : |v| = n\}$ (nth generation).
$o \leftrightarrow v$ - vertices/edges on geodesic connecting o and v.

BRW model

$\{X_e\}_{e \in E}$ i.i.d., law μ.
$S_v = \sum_{e \in o \leftrightarrow v} X_e$ sum along geodesic (BRW).

Maximal displacement:

$$M_n = \max_{v \in D_n} S_v .$$

Assumptions (for these lectures):

μ possesses super-exponential tails:

$$E_\mu(e^{\lambda X_e}) =: e^{\Lambda(\lambda)} < \infty, \quad \lambda \in \mathbb{R}.$$

The tree $\mathcal{T}$ is a k-ary tree, with $k \geq 2$: $d_v = k + 1_{v \neq o}$.